$$\delta_1 = \frac{A^2 - 1}{2a} + 8\,\frac{Bu_f}{Bo_f}\,(B^2 - 1)\left(1 - Bu_f \tan^{-1}\frac{1}{Bu_f}\right) + \ldots \tag{18d}$$

The nature of the weak roots is clearly shown by Eqs. (18). There are, in each case, two different kinds of terms that give the effect of radiation as well as relaxation on the waves. In δ_1, the last term for slow oscillations (a, $b \ll 1$) is the equilibrium δ_l, and for fast oscillations (a, $b \gg 1$) is the frozen δ_l [see Eq. (16a)]. It can be shown that the relaxation causes δ to have a maximum value of $\delta_1^{(0)} = 2^{-3/2}(A^2 - 1)(1 + A^2)^{-1/2}$ at $a^2 = (1 + 3A^2)(A^2 + 3)^{-1}$, i.e., at ultrasonic frequencies. Previously we found that the radiation also caused δ_l to have a maximum. Therefore, there are in general two peaks in the ω spectrum of δ_1, and the relative magnitude of the peaks depends on the thermal state of the gas. Regarding the strong roots, similar results such as those found in the equilibrium case are obtained, but λ_2, δ_2, and $Bu_f{}^*$ are much more complicated functions of the various parameters.

References

[1] Ryhming, I. L., "Effects of non-grey radiation on wave propagation in a simple dissociating gas," Aerospace Corp., El Segundo, Calif., TDR-269(4230-20)-2 (January 13, 1964).

[2] Vincenti, W. G., "Nonequilibrium flow over a wavy wall," J. Fluid Mech. **6**, 481–496(1959).

[3] Kourganoff, V., *Basic Methods in Transfer Problems* (Dover Publications, Inc., New York, 1963), p. 31.

[4] Sulzer, P. and Wieland, K., "Intensitätsverteilung eines Kontinuerlichen Absorptionspectrums in Abhängigkeit von Temperatur und Wellenzahl," Helv. Phys. Acta **25**, 653–676 (1952).

[5] Vincenti, W. G. and Baldwin, B. S., Jr., "Effect of thermal radiation on the propagation of plane acoustic waves," J. Fluid Mech. **12**, 449–477(1962).

Experiment in Solar Orientation of Spin Stabilized Satellite

M. C. Crocker II* and E. A. Vrablik*
Massachusetts Institute of Technology,
Lexington, Mass.

Early in 1965, Massachusetts Institute of Technology (MIT) Lincoln Laboratory will orbit an experimental satellite designed to test communications techniques and components. The satellite will be spinning at 180 rpm, and, in order to equalize the satellite skin temperatures and increase solar cell efficiency, an attempt will be made to precess the satellite's spin vector perpendicular to the solar vector and maintain it in that position, using a magnetic torquing system that requires no commands from the ground. The advantage of this orientation to a high area thermal-mass ratio satellite with no batteries aboard is a temperature balance more favorable to power conversion. The orientation will be accomplished by controlling the magnitude and direction of the satellite's magnetic moment, which will be aligned with the largest principal moment of inertial ($\cong$1.5 kg-m^2) of the satellite (coincident with the spin axis). This magnetic moment will interact with the earth's geomagnetic field to produce a controlled orientation torque.

Received February 25, 1965.

* Staff Member, Lincoln Laboratory, which is operated with support from the U. S. Air Force.

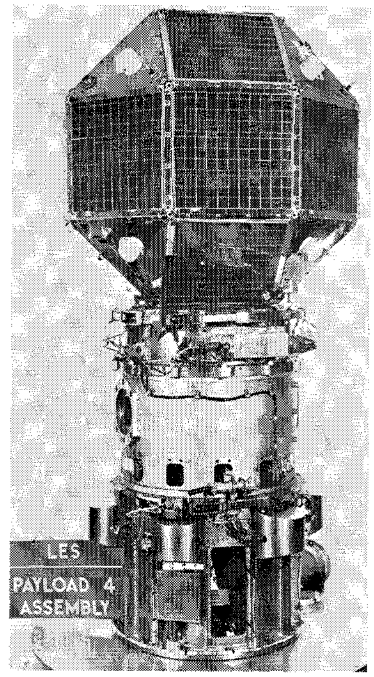

Fig. 1 LES payload 4 assembly.

Construction of a Torquing Device for the Lincoln Experimental Satellite (LES)

The LES is in the form of a 26-sided polyhedron (Fig. 1) with solar cell panels covering all of the square faces. The satellite has a total of eighteen solar panels, eight in an equatorial belt around the spin axis and five above and five below this equatorial belt.

All the solar panels above the satellite equatorial belt and four alternate panels on the belt are connected in parallel. The total output current from these panels is designated I upper. The remaining panels are connected in parallel, and their total current is designated I lower. If these currents are routed in opposite directions in multiturn air-core coils to produce a magnetic moment proportional to their difference, this difference current ΔI can be written in vector form as the dot product of the unit vector along the spin axis **S** and the sun vector **SOL**:

$$\Delta I = I_{\text{upper}} - I_{\text{lower}} = I\mathbf{S}\cdot\mathbf{SOL} \tag{1}$$

The magnetic moment **M** produced by this difference current is proportional to the area of the coils A and the number of turns N in each coil and is directed along **S**:

$$\mathbf{M} = NA\Delta I\mathbf{S} = NAI(\mathbf{S}\cdot\mathbf{SOL})\mathbf{S} \tag{2}$$

Since these coils are in series with the solar panels and the main satellite power bus, it is necessary that the resistance of these coils be low, or all the solar panel current will be turned to heat in these coils, and there will be no power left for the rest of the satellite to function. The present solar cells produce 16 v and 1.4 amp, nominally. In order to limit the coil power consumption to 1% of the total, their voltage drop must not exceed 0.16 v when all 1.4 amp flow through one coil. Thus, the coil resistance is limited to about 0.12 ohm. The total satellite weight is only about 50 lb, and only 1 lb is to be used for the orientation experiment. The area of the coils is limited to about π ft^2 or 0.29 m^2 by the size of the satellite (24 in. across faces). These restrictions on weight, resistance, and coil size restrict the number of turns to about five (with aluminum wire). Hence, the maximum magnetic moment that can be created with an air-core system is $M_{\max}$:

$$M_{\max} = NA\Delta I_{\max} \cong 5(0.29)1.4 \cong 2.0 \text{ amp-m}^2$$

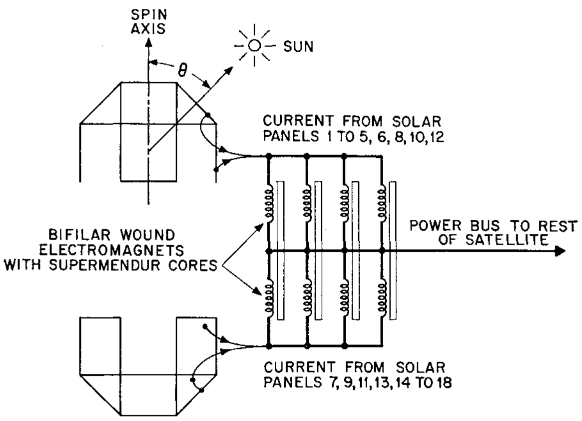

Fig. 2 LES magnetic orientation system.

However, if a core of very soft magnetic material with high intrinsic magnetization (such as Supermendur) is used instead of air cores, $M_{\max}$ can be increased with no increase in power consumption or weight. Four electromagnetic coils with Supermendur cores 0.114 in. in diameter and 23 in. long, properly wound and spaced, theoretically can produce a maximum magnetic moment of 29-amp-m^2, and values of 18-amp-m^2 have been attained (the discrepancy is due to the difficulties in manufacturing Supermendur in this shape).

With the Supermendur cores, the variation of magnetic moment is no longer specified by Eq. (2), but ΔI is still given by Eq. (1). Instead of Eq. (2), we find that $\mathbf{M}$ is multivalued and depends nonlinearly on ΔI. The value of $\mathbf{M}$ at any instant is found from the $B - H$ curve of Supermendur and the past values of ΔI.

A first approximation for the dependence of magnetic moment on coil current is to assume that Supermendur has a perfectly square $B - H$ curve that has no hysteresis loss, and switches from $+B_{\text{sat}}$ to $-B_{\text{sat}}$ as H changes sign. Then the variation of magnetic moment with orientation becomes

$$\mathbf{M} = +M_{\text{sat}}\mathbf{S} \qquad \mathbf{S}\cdot\mathbf{SOL} \geq 0 - M_{\text{sat}}\mathbf{S} \qquad \mathbf{S}\cdot\mathbf{SOL} < 0 \quad (3)$$

A schematic picture of the electromagnet arrangement is shown in Fig. 2.

The torque $\mathbf{T}$, produced by the interaction of this magnetic moment with the earth's geomagnetic field $\mathbf{B}$, is

$$\mathbf{T} = \dot{\mathbf{J}} = \mathbf{M} \times \mathbf{B}$$

where $\mathbf{J}$ is the satellite angular momentum. In the "fast-top" approximation, we may solve this equation by directing the angular momentum along the spin axis at all times, thus, $= |\mathbf{J}|\mathbf{S}$ and

$$\mathbf{T} = \dot{\mathbf{J}} = |\mathbf{J}|\dot{\mathbf{S}} = \mathbf{M} \times \mathbf{B} = \mp M_{\max}(\mathbf{S} \times \mathbf{B}) \qquad \mathbf{S}\cdot\mathbf{SOL} \gtrless 0$$

The solution of this equation is characterized by a precession of the spin axis $\mathbf{S}$ around the instantaneous value of $\mathbf{B}$, with the details of the motion dependent on $\mathbf{B}$ and the solar vector. A 7094 computer program was written to calculate the value of $\mathbf{B}$ (using a 11.4° tilted dipole approximation for the earth's field) as the satellite moves in its orbit; the solar vector; and the vector relationships necessary to perform a numerical integration of the spin-axis motion, using Eq. (3) to specify the magnetic moment.

The results of this program indicate that the satellite to be launched early in 1965 with 1.5K- to 10.0K-naut-mile orbit inclined 32° to the equator (eccentricity $\cong$ 0.53, period $\cong$ 7.5 hr) will experience a maximum precessional rate of about 5°/day around the average magnetic-field vector (which will be inclined about 23° to the orbit normal). Thus, if the satellite were to be launched so that the spin vector pointed into (or away from) the sun initially, it would precess to perpendicularity in about 20 days and maintain this orientation thereafter, as the earth and satellite orbit the sun.

Newton-Raphson Operator; Problems with Undetermined End Points

Robert S. Long*
University of Canterbury, Christchurch, New Zealand

In a recent paper, McGill and Kenneth[1] have successfully applied the technique of quasi-linearization (a generalized Newton-Raphson operator) to the numerical solution of certain optimization problems. These involve determining the solution of a set of nonlinear differential equations with boundary conditions specified at two points, i.e., at two values of the independent variable t, namely $t = 0$ and $t = t_f$. However, in one of their examples, the value of t_f is unknown, and so must be determined as part of the whole iterative process.

In the paper quoted, the method given for dealing with t_f appears to suffer from two disadvantages. First, it is not an integral part of the quasi-linearization process. In what may be called a normal quasi-linearization process, all the boundary conditions are exactly satisfied at each iteration (apart from roundoff and truncation error). Presumably it is this feature of a normal process which helps to explain its excellent convergence. In the method given, however, one of the boundary conditions is used to modify t_f, and this boundary condition is not satisfied until convergence has actually occurred. This feature may be expected to slow down the rate of convergence. Second, the method given depends on the numerical calculation of a derivative, a procedure to be avoided if possible.

The purpose of this note is to present an alternative method for dealing with t_f which avoids both of the forementioned disadvantages. We make a change of variable by writing $t = as$ where s is the new independent variable, and a is a constant to be determined. We take the end points to be $s = 0, s = 1$, so that, once a is found, the value of t_f is given by $t_f = a$. Suppose that a typical differential equation of the system is $\dot{x} = f$, where f is, in general, a function of all the dependent variables, and also of t, and $\dot{x} = dx/dt$. Writing $dx/ds = x'$, we have $x' = (dx/dt)(dt/ds) = a\dot{x}$ so that $x' = af$ where, in f, t is replaced by as. We now treat a as an additional dependent, or state, variable, and include it in the quasi-linearization process. Then the set of equations is of exactly the same form as before, and there will be the correct number of boundary conditions. The equations are slightly more complicated than before, but not unduly so. Of course, a is a constant during each iteration, but, like the other state variables, it varies from one iteration to the next. This treatment of a as a state variable may be formalized by writing $a' = 0$ and by adding this equation to the system, but, from a computational point of view, this is not necessary.

Because of the excellent convergency properties of the quasi-linearization process, and because the determination of t_f is now an integral part of that process, it is to be expected that the total computation time will be reduced as compared with the time for the previous method. This, however, is a matter for numerical experiment.

Received February 15, 1965; revision received March 29, 1965.

* Senior Lecturer, Department of Mathematics.